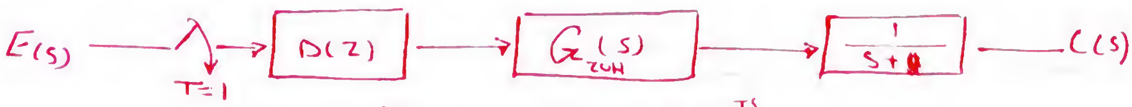
Sheet 2

2 Find Pulse Transfer function of



$D(z) = \frac{z}{z-1}$ $\quad G_{ZOH}(s) = \frac{1-e^{-Ts}}{s}$

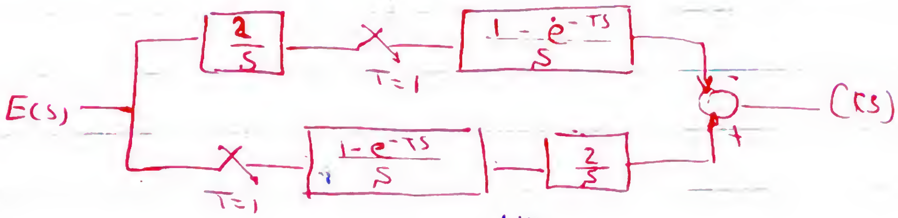
Sol^n

$$\frac{C(z)}{D(z)} = D(z)\, Z\left[\frac{1-e^{-Ts}}{s(s+1)}\right] = \frac{z}{z-1}\cdot(1-z^{-1})\, Z\left[\frac{1}{s(s+1)}\right]$$

$$= Z\left[\frac{1}{s} - \frac{1}{s+1}\right] = Z\left[u(k) - e^{-k}\right]$$

$$= \frac{z}{z-1} - \frac{z}{z-e^{-1}} = \frac{z(z-0.368) - z(z-1)}{(z-1)(z-0.368)} = \boxed{\frac{0.632\, z}{(z-1)(z-0.368)}}$$

4 Find the output C(k) for e(t) equal to unit step



Sol^n

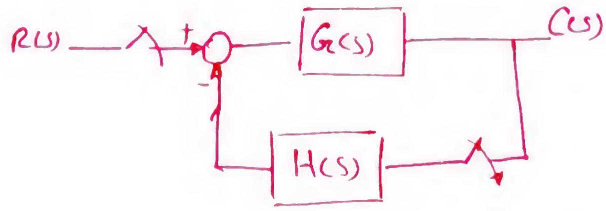
$$C(z) = E(z)\, Z\left[\frac{2(1-e^{-Ts})}{s^2}\right] - Z\left[\frac{1-e^{-Ts}}{s}\right] Z\Big[\frac{2}{s}\cdot \underbrace{E(s)}_{=\frac{1}{s}}\Big]$$

unit step

$$= \cancel{\frac{z}{z-1}}\cdot\cancel{(1-z^{-1})}\, Z\left[\frac{2}{s^2}\right] - (1-z^{-1})\, \underbrace{Z\left[\frac{1}{s}\right]}_{=\frac{z}{z-1}}\, Z\left[\frac{2}{s^2}\right]$$

$$= \frac{2z}{(z-1)^2} - \frac{2z}{(z-1)^2} = \text{zero}$$

Find $C(z)$



Sol^n

$$C(s) = G(s)\,E(s) \quad \text{---(1)}$$

$$E(s) = R^*(s) - H(s)\,C^*(s) \quad \text{---(2)}$$

$$\therefore C(s) = G(s)\left[R^*(s) - H(s)\,C^*(s)\right] \quad // \text{ substitute by 2 in 1}$$

$$C(s) = G(s)\,R^*(s) - GH(s)\,C^*(s) \quad \text{---(3)}$$

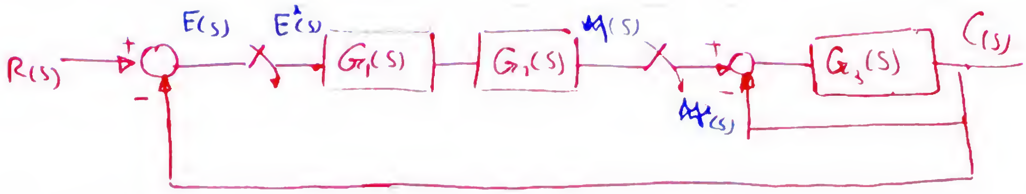
staring

$$C^*(s) = G^*(s)\,R^*(s) - \overline{GH}^*(s)\,C^*(s) \quad // \text{ staring of 3}$$

$$C^*(s)\left(1 + \overline{GH}^*(s)\right) = G^*(s)\,R^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH}^*(s)} \longrightarrow \frac{C(z)}{R(z)} = \frac{G(z)}{1 + \overline{GH}(z)}$$

Find $\dfrac{C(z)}{R(z)}$



Sol^n

$$C(s) = G_3(s)\left[M^*(s) - C(s)\right]$$

$$\left(1 + G_3(s)\right) C(s) = G_3(s)\,M^*(s)$$

$$\therefore C(s) = \frac{G_3(s)}{1 + G_3(s)}\,M^*(s)$$

But we have

$$M(s) = G_1G_2(s)\,E^*(s) = G_1G_2(s)\left(R(s) - C(s)\right)^*$$

$$= G_1G_2(s)\left(R^*(s) - C^*(s)\right)$$

Staring of M

$$M^*(s) = \overline{G_1G_2}^*(s)\left(R^*(s) - C^*(s)\right)$$

$$\therefore C(s) = \frac{G_3(s)}{1 + G_3(s)}\,\overline{G_1G_2}^*(s)\left(R^*(s) - C^*(s)\right)$$

staring above eqn

$$C^*(s) = \left[\frac{G_3(s)}{1 + G_3(s)}\right]^* \overline{G_1G_2}^*(s)\left(R^*(s) - C^*(s)\right)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{\left[\dfrac{G_3(s)}{1 + G_3(s)}\right]^* \overline{G_1G_2}^*(s)}{1 + \left[\dfrac{G_3(s)}{1 + G_3(s)}\right]^* \overline{G_1G_2}^*(s)}$$

Note
we first substitute then star the eqn